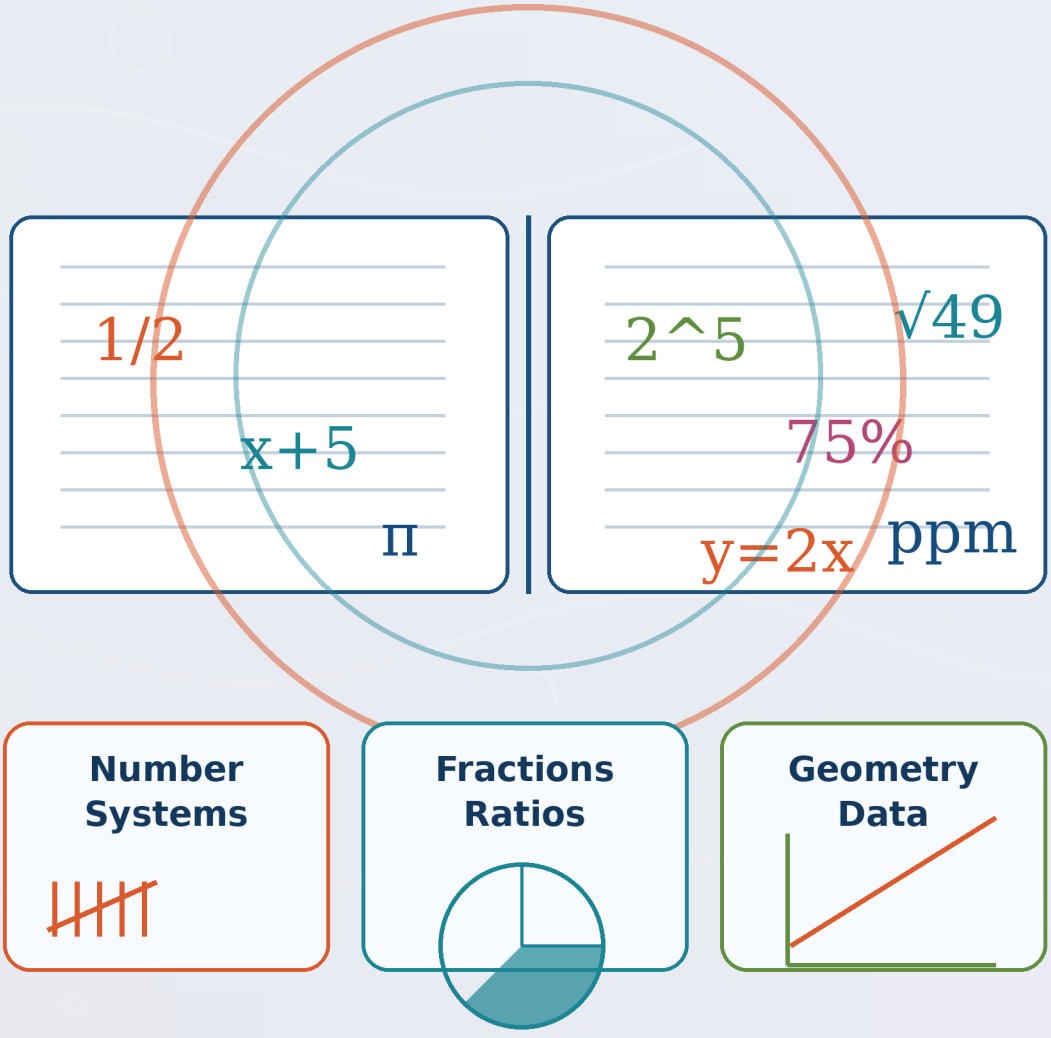
Foundations of Mathematics

Volume 1

A Visual Problem-Solving Book for Ages 9-13



Based on the First 50 Free YouTube Live Classes (2023-24)

Dr. Biplab Pal

Mathematics means simplification through better questions.

Foreword

This rebuilt English edition is designed as a true workbook-textbook for children aged 9-13. It follows the spirit of the first 50 free online YouTube Live mathematics classes taught during 2023-24. Volume 1 keeps the same theme: mathematics grows from human needs, becomes powerful through symbols, and becomes useful when students learn how to ask better questions.

The new version is intentionally problem-centered. Every chapter now has visual explanations, theory, solved classroom examples, video links, and exactly 30 graded exercises: 10 easy, 10 classroom-level and 10 super-challenge. The hard problems are meant to be discussed, drawn, paused, restarted and solved with hints. They are not meant to be finished by guessing.

Teaching philosophy. Mathematics means simplification. A difficult story becomes easier when we name the unknown, draw a diagram, choose units, build an equation, and check whether the answer makes sense.

How students should use this book

1. Read the visual explanation first. Redraw the diagram in your notebook.
2. Watch or re-watch the linked class video.
3. Study the solved examples and cover the solution before trying again.
4. Solve the 10 easy problems without help.
5. Attempt the 10 classroom problems with diagrams and unit checks.
6. Try the 10 super-challenge problems slowly. Use hints only after a serious attempt.
7. For every hard problem, write one sentence explaining the idea, not only the answer.

Problem levels

Level	Purpose

Easy 1-10	Build confidence and check the basic idea. These should be possible after reading the theory.
Classroom 11-20	Match the normal age range, but require careful reading, units and multi-step thinking.
Super Challenge 21-30	Thought-provoking problems for discussion, drawing, trial, proof and deeper reasoning. They are meant to stretch strong students.

Main video path

The primary video path is the free playlist Math Fundamentals with Dr. Biplab Pal. Each chapter also contains supporting links from Dr. Pal's classes or other visual-learning resources.

Using AI assistants wisely

AI systems such as ChatGPT, Gemini and other learning assistants can help you understand an idea, check a mistake, create another example or ask you practice questions. They should not replace your own thinking. In this book, each chapter gives safe student prompts. A good rule is: ask for a hint first, then try, then ask the assistant to check your reasoning.

The 4-step AI learning rule: (1) State the topic. (2) Ask for a picture, table or story. (3) Ask for hints before answers. (4) Explain your own solution and ask the AI to find only the first mistake.

Contents

Chapter 1

Numbers: From Counting Marks to Zero and Binary

Classroom question: Why did humans need better number systems, and why did zero change everything?

Learning goals

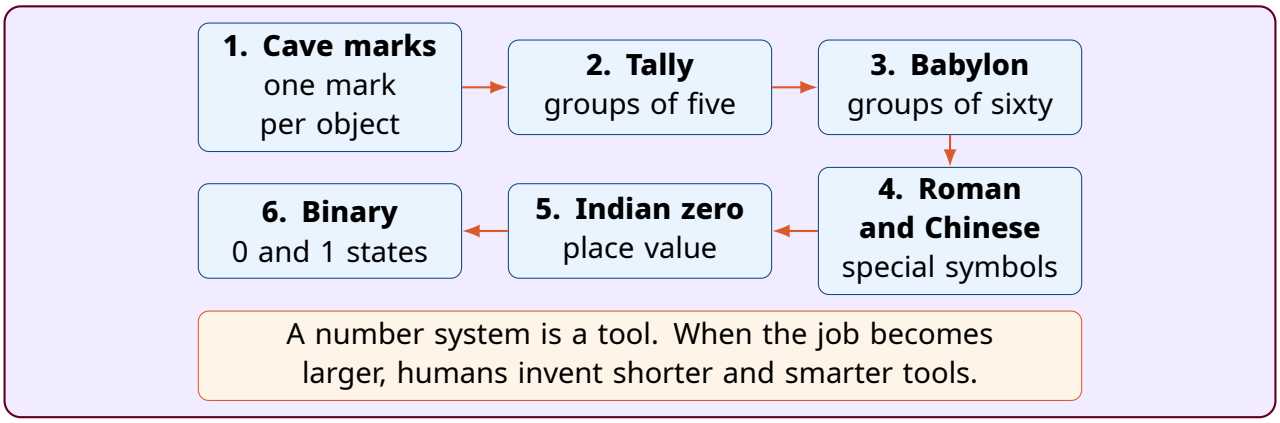
- Separate quantity, numeral and number system.
- Use place value to read and build numbers.
- Compare tally, Babylonian, Roman, Indian decimal and binary systems.
- Convert small numbers between decimal and binary.

Theory and explanation basics

A number is an idea; a numeral is the written form of that idea. Thirty-five fish, the symbol 35, the Roman form XXXV and the binary form 100011 can all represent the same quantity. The power of a number system is measured by how easily it lets people write, compare and calculate.

The Indian decimal system became powerful because the same ten digits can represent huge quantities by position. Zero has two jobs: it can mean no objects, and it can hold an empty place. In 507, the 0 says there are no tens; without it, 57 would mean a different number. Binary uses only 0 and 1, which is perfect for machines with two stable states.

Visual idea



Watch and learn further

- Dr. Pal playlist
- Dr. Pal - History of numerals
- Dr. Pal - Large and small numbers
- Numberphile - Number systems

Two groups of online learning links

Group A: Dr. Biplab Pal class links

- Full Dr. Pal playlist - start from the first 50 classes
- Dr. Pal - History of numerals / origin of numbers
- Dr. Pal - How to use Math?
- Dr. Pal - LCM/HCF and number structure

Group B: Other YouTube references

- Numberphile - The change from Roman numerals
- Numberphile - Binary-style number curiosity
- YouTube search - Khan Academy place value
- YouTube search - Math Antics place value

AI prompts for this chapter

Use these prompts after trying the ideas yourself. Ask for hints before full answers.

1. *"Explain place value to a 10-year-old using fish, seeds and empty boxes for zero."*
2. *"Give me only a hint for converting a decimal number to binary; do not solve it completely."*
3. *"Make a small table comparing tally, Roman, decimal and binary for the numbers 1 to 12."*
4. *"Ask me five questions, one at a time, about why zero is useful as a placeholder."*
5. *"Here is my answer for a binary conversion: [paste my work]. Find the first mistake only."*

Solved classroom examples

Place value with zero
Write 3048 as expanded form.
$3048 = 3 \times 1000 + 0 \times 100 + 4 \times 10 + 8 \times 1$. The zero is not useless; it preserves the hundreds place.

Convert 67 to binary
The powers of 2 are 64, 32, 16, 8, 4, 2, 1. Since $67 = 64 + 2 + 1$, the binary digits are 1000011_2.

Babylonian grouping
In base 60, $143 = 2 \times 60 + 23$, so it can be read as two groups of sixty and twenty-three more. This is why time language such as hours and minutes still feels base-60.

Exercises: 30 problems with hints

Easy practice: Problems 1-10

1. Write 48 in expanded form using hundreds, tens and ones.
Hint: Separate the tens and ones: 48 = 4 tens + 8 ones.

Your thinking space:

2. Write 305 as expanded form and explain the role of 0.
Hint: Ask what the tens digit means when no tens are present.

Your thinking space:

3. Make tally marks for 27 using groups of five.
Hint: Use five full groups and two extra marks.

Your thinking space:

4. Convert 9, 10 and 11 into binary.
Hint: Use powers 8, 4, 2, 1.

Your thinking space:

5. Write 76 as one group of 60 plus a remainder.
Hint: Divide 76 by 60 and keep the remainder.

Your thinking space:

6. Which is larger: 1001 or 999? Explain using place value.
Hint: A four-digit number with a thousand is larger than any three-digit number.

Your thinking space:

7. Write 29 in Roman numerals.
Hint: Use XX for 20 and IX for 9.

Your thinking space:

8. What decimal number is 1010_2?
Hint: Use 8, 4, 2, 1 as place values.

Your thinking space:

9. Why are ten fingers connected to base 10?
Hint: Think about how early counting may have used hands.

Your thinking space:

10. Create a three-symbol code for 0, 1 and 2, then write the numbers 0 to 8 in base 3.
Hint: Use places 1, 3 and 9.

Your thinking space:

Classroom practice: Problems 11-20

11. A village clerk counts 156 sacks of rice using tally groups of five. How many full groups of five and how many extra marks are needed?
Hint: Divide 156 by 5.

Your thinking space:

12. Write 2026 as expanded form. Then write the value of each digit separately.
Hint: Use thousands, hundreds, tens and ones.

Your thinking space:

13. Convert 11011_2 to decimal and explain each switched-on place.
Hint: Use 16, 8, 4, 2, 1.

Your thinking space:

14. Convert 45 to binary by subtracting powers of 2.
Hint: Start with 32, then use what remains.

Your thinking space:

15. A Babylonian-style clock has 2 hours and 35 minutes. Convert it to total minutes.
Hint: Each hour is 60 minutes.

Your thinking space:

16. Why is MMXXIV less useful for multiplication than 2024?
Hint: Compare place value with repeated Roman symbols.

Your thinking space:

17. Write a rule for reading any four-digit decimal number.

Hint: State thousands, hundreds, tens and ones.

Your thinking space:

18. A computer byte has 8 binary places. If all places are 1, what decimal number is stored?
Hint: Add 1+2+4+8+16+32+64+128.

Your thinking space:

19. A student writes 67 as 60 and 7 but says it is base 10. What is missing in this explanation?
Hint: The grouping is base-60 style; decimal uses tens and ones.

Your thinking space:

20. Compare 31, 32 and 33 in binary. What pattern appears when crossing a power of 2?
Hint: Convert all three and look at the length of the string.

Your thinking space:

Super challenge: Problems 21-30

21. Invent a number system for a planet whose people have six fingers. Write decimal 100 in that system and explain each place.
Hint: Use powers 6, 36 and 216.

Your thinking space:

22. A stone tablet has a smudged zero-like space. Explain how 106, 1006 and 16 could

be confused if the placeholder is lost.
Hint: List their place values and identify empty positions.

Your thinking space:

23. A store wants the shortest labels for 1000 items. Compare decimal, binary and base 60 by number of positions needed.
Hint: Find or estimate representations in each base.

Your thinking space:

24. Design a tally-plus-place-value hybrid system for counting up to 10,000. Explain why it is better than plain tally.

Hint: Create small groups and large groups.

Your thinking space:

25. A message uses only on/off lamps. How many different numbers can be shown with 5 lamps? What is the largest number if 00000 means zero?
Hint: Think of each lamp as a binary digit.

Your thinking space:

26. Explain in a paragraph why zero was harder to invent than the symbol 7.
Hint: Distinguish counting objects from naming an empty place.

Your thinking space:

27. Create a fair test comparing Roman and decimal notation for adding 487 + 659. Which steps are easier in decimal?
Hint: Try expanding both numbers by place value.

Your thinking space:

28. A time machine student sees 3 tally systems: groups of 4, 5 and 10. Which is most efficient for 247? Justify with number of groups and leftovers.
Hint: Compute full groups and leftover marks.

Your thinking space:

29. A code writes decimal numbers using base 2 but reads from right to left by mistake. Find a number whose mistaken reading changes its value and explain.
Hint: Try 1010 or 110.

Your thinking space:

30. Prove that any 3-digit decimal number abc equals $100a + 10b + c$. Use a specific example and then the general idea.
Hint: Connect position to powers of ten.

Your thinking space:

Notebook reflection. Which problem forced you to draw a diagram? Which problem had the most surprising idea? Write one mistake you made and how you fixed it.

Chapter 2

Word Problems and Mathematical Translation

Classroom question: How do we turn a story into symbols without losing meaning?

Learning goals

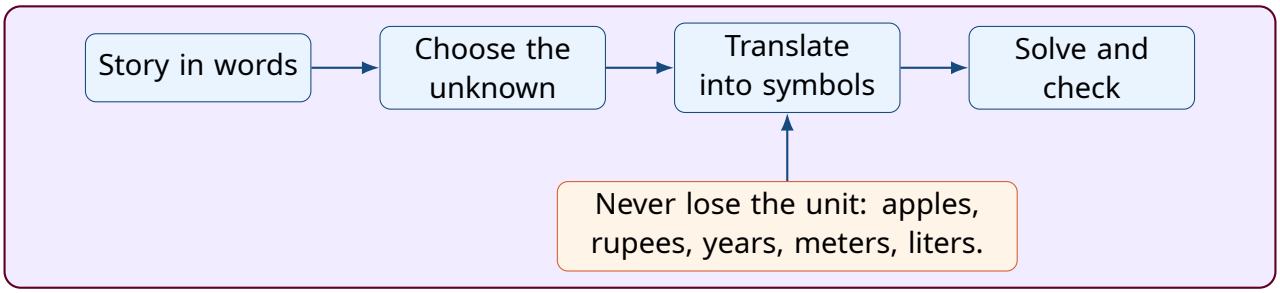
- Identify the unknown and choose a useful variable.
- Translate words such as total, difference, of and per.
- Track units carefully.
- Recognize when information is missing or irrelevant.

Theory and explanation basics

Most students lose word problems before the calculation begins. The important work is translation. First find what is being asked. Then name the unknown. Then turn relationships into symbols. The equal sign means two descriptions of the same quantity, not just an instruction to compute.

Units protect meaning. Six apples plus six oranges is twelve fruits, not twelve apples. A rate such as 60 km per hour connects distance and time. A good equation can be read back into the original story. If the sentence sounds wrong when you read it back, the equation is probably wrong.

Visual idea



Watch and learn further

- Dr. Pal playlist
- Dr. Pal - Word Problem-2, Class 3
- Dr. Pal - How to use Math?
- Math Antics playlists

Two groups of online learning links

Group A: Dr. Biplab Pal class links

- Full Dr. Pal playlist - word problems appear across the class sequence
- Dr. Pal - Word Problem-2, Class 3
- Dr. Pal - Word Problem and Algebra
- Dr. Pal - Problem Solving with one variable

Group B: Other YouTube references

- YouTube search - Khan Academy word problems
- YouTube search - Math Antics word problems
- YouTube search - Tape diagrams for word problems
- YouTube search - Bar model word problems

AI prompts for this chapter

Use these prompts after trying the ideas yourself. Ask for hints before full answers.

1. *"Turn this story problem into a picture, a table and an equation, but do not solve yet: [paste problem]."*
2. *"Help me identify the unknown in this word problem and suggest a good variable name."*
3. *"Give me three possible diagrams for this problem and explain which is easiest for a 12-year-old."*
4. *"Ask me guiding questions to solve this problem step by step, without giving the final answer."*
5. *"Check if my equation matches the words. My equation is [paste equation] and the story is [paste story]."*

Solved classroom examples

Age translation
Shreya is 8 years older than Shyam. Let Shyam be x. Then Shreya is $x + 8$. Next year: $x + 8 + 1 = 2(x + 1)$. Solve: $x + 9 = 2x + 2$, so $x = 7$. Shyam is 7 and Shreya is 15.

Unit check
A bottle has 2 liters of juice. Each glass holds 250 mL. Convert 2 liters to 2000 mL first. Then $2000 \div 250 = 8$ glasses.

Missing information
If a ship has 20 goats and 14 cows, find the captainś age. This is not solvable because animal counts have no given relation to age.

Exercises: 30 problems with hints

Easy practice: Problems 1-10

1. Translate: A number increased by 7 is 18.
Hint: Let the number be x.

Your thinking space:

2. Translate: Three times a number is 24.
Hint: Use multiplication and an equal sign.

Your thinking space:

3. If Rina has 5 apples and buys 8 more, how many apples does she have?
Hint: Same unit, so add.

Your thinking space:

4. A pen costs Rs. 12. What is the cost of 6 pens?
Hint: Cost per pen times number of pens.

Your thinking space:

5. Translate: A number divided by 4 leaves 9.
Hint: Use x/4 = 9.

Your thinking space:

6. A bus travels 60 km in 2 hours. What is the speed?
Hint: Speed = distance/time.

Your thinking space:

7. Write a question that cannot be solved because information is missing.
Hint: Make sure the unknown is not connected to the data.

Your thinking space:

8. In the phrase ”5 less than a number”, which comes first in the expression?
Hint: Less than reverses order: x - 5.

Your thinking space:

9. A classroom has 4 rows with 6 desks each. How many desks are there?
Hint: Repeated equal rows mean multiplication.

Your thinking space:

10. Translate: the total of x and 13 is 40.
Hint: Total often means addition.

Your thinking space:

Classroom practice: Problems 11-20

11. A father and son are together 48 years old. The father is 30 years older. Find their ages.
Hint: Let the son be x and the father be x+30.

Your thinking space:

12. A book has two parts. Part 1 has 40 more pages than Part 2. If total pages are 220, find each part.
Hint: Let Part 2 be x.

Your thinking space:

13. A dress is discounted by 20% from Rs. 800. Write the calculation in two ways: percentage and fraction.
Hint: 20% = 1/5.

Your thinking space:

14. A jar has red and blue marbles. There are 3 more red than blue and 17 total. Find

each color.
Hint: Let blue be x.

Your thinking space:

15. A car travels at 50 km/h for some hours and covers 200 km. How long did it travel?
Hint: Time = distance/speed.

Your thinking space:

16. A student writes 3(x+5) as 3x+5. Explain the mistake with a story.
Hint: Three groups of x+5 means three x and three 5.

Your thinking space:

17. A farmer has chickens and goats, 10 animals and 28 legs. Write equations for number of animals and legs.
Hint: Use c and g; chickens have 2 legs, goats have 4.

Your thinking space:

18. A school buys notebooks at Rs. 25 each and pencils at Rs. 5 each. It buys 12 items for Rs. 180. Write equations.
Hint: Use n+p=12 and cost equation.

Your thinking space:

19. An elevator starts at floor 3, goes up 8 floors, down 5 floors, then up 2 floors. Where is it?
Hint: Represent up as positive and down as negative.

Your thinking space:

20. A number puzzle says: double a number, add 9, and get 31. Solve and check by reading the story backward.
Hint: Build 2x+9=31.

Your thinking space:

Super challenge: Problems 21-30

21. The Alien Zoo has Trilogs with 3 legs, Quadrogs with 4 legs and Penterogs with 5 legs. The ratio of Trilogs to Quadrogs is 2:3, and Quadrogs to Penterogs is 4:5. Find a possible smallest group and total legs.
Hint: Match the Quadrogs by using common multiples.

Your thinking space:

22. A bookstore sells novels at Rs. 800 and comics at Rs. 300. Ravi buys 7 books for Rs. 4100. Find the numbers of each type.
Hint: Use two equations or try possible novel counts.

Your thinking space:

23. A number leaves remainder 4 when divided by 7 and remainder 2 when divided by 3. Find the number between 1 and 21.
Hint: List numbers 4 more than multiples of 7.

Your thinking space:

24. Create two different stories that both translate to $4x+12=40$. Explain why the same equation can model different situations.
Hint: Use money in one story and distance in another.

Your thinking space:

25. A student solves a problem and gets 3.5 children. Explain why the equation might be mathematically correct but the interpretation impossible.
Hint: Some quantities must be whole numbers.

Your thinking space:

26. The Theater Seats problem: 25 rows would gain 300 seats total if each row had more seats. How many extra seats per row is that? Then generalize.
Hint: Divide the total extra seats by rows.

Your thinking space:

27. A recipe uses cups and milliliters in the same problem. Design a wrong solution and then fix it by converting units.
Hint: Use 1 cup = about 240 mL or state your conversion.

Your thinking space:

28. A bus ticket costs twice as much as a train ticket. Three bus and two train tickets

cost Rs. 400. Find both prices.
Hint: Let train = x, bus = 2x.

Your thinking space:

29. A problem gives a lot of extra data about colors, dates and names. Write a method for ignoring irrelevant data safely.
Hint: Identify the unknown first, then circle only linked quantities.

Your thinking space:

30. Build a two-step word problem whose answer is 42 but where a careless student might answer 40. Explain the trap.

Hint: Use ”less than” or a hidden unit conversion.

Your thinking space:

Notebook reflection. Which problem forced you to draw a diagram? Which problem had the most surprising idea? Write one mistake you made and how you fixed it.

Chapter 3

Fractions, Percentages, PPM and PPB

Classroom question: How can one mathematical language describe pizza, discount, pollution and tiny scientific quantities?

Learning goals

- Represent fractions using area, length and division.
- Convert fractions, decimals and percentages.
- Use ppm and ppb for very small fractions.
- Solve multi-step fraction and percent problems with units.

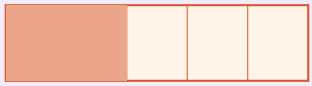
Theory and explanation basics

A fraction names a part of a whole. The denominator tells how many equal parts the whole is cut into; the numerator tells how many of those parts are selected. Fractions are not only pizza pieces. They also describe tax, discount, interest, medicine concentration and scientific measurements.

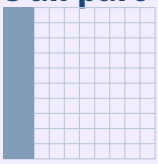
Percent means per hundred. It is useful because many people can compare 20%, 35% and 70% quickly. PPM means parts per million, and PPB means parts per billion. These units are fractions written for tiny concentrations. For water, 1 ppm is often treated as about 1 milligram per liter when the solution is dilute.

Visual idea

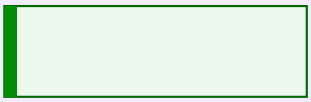
Fraction, percent and ppm are all part-of-whole languages



$2/5 = 40\%$



20 of 100 squares = 20%



A very tiny part may be described as ppm or ppb.

Watch and learn further

- Dr. Pal playlist
- Khan Academy - understand fractions
- Khan Academy - percents from fraction models
- Math Antics - percentages

Two groups of online learning links

Group A: Dr. Biplab Pal class links

- Full Dr. Pal playlist - fraction and percent ideas
- Dr. Pal - Small Fraction
- Dr. Pal - Inflation percent
- Dr. Pal - Sense of reference

Group B: Other YouTube references

- Khan Academy YouTube - Fractions, decimals and percentages
- Math Antics YouTube - Fractions playlist
- YouTube search - ppm ppb explained for students
- YouTube search - percentage word problems for grade 6

AI prompts for this chapter

Use these prompts after trying the ideas yourself. Ask for hints before full answers.

1. *"Explain fractions as sharing, measuring and comparing. Use a pizza, a ruler and a classroom vote."*
2. *"Show me how 3/4, 0.75 and 75 percent are connected using a diagram."*
3. *"Explain ppm and ppb using a water tank story for a 12-year-old."*
4. *"Give me a hint for this percentage problem, then wait for my attempt: [paste problem]."*
5. *"Create one easy, one medium and one hard practice problem on fractions and percentages, with hints only."*

Solved classroom examples

Chocolate sharing
Tom eats $2/5$ of a bar and Jerry eats $3/10$. Convert to tenths: $2/5 = 4/10$. Total eaten is $7/10$, so $3/10$ remains.

Discount as percent and fraction
A Rs. 800 dress has 20% off. Since $20\% = 1/5$, the saving is $800 \times 1/5 = 160$. New price: Rs. 640.

PPM tank problem
A tank has 2700 liters of water with 400 ppm mud. Mud amount $= (400/1{,}000{,}000) \times 2700 = 1.08$ liters approximately.

Exercises: 30 problems with hints

Easy practice: Problems 1-10

1. Shade $3/8$ of a rectangle divided into 8 equal boxes.
Hint: Shade 3 boxes and leave 5 unshaded.

Your thinking space:

2. Convert $1/4$ to a percent.
Hint: Make denominator 100 or divide 1 by 4.

Your thinking space:

3. Which is larger: $2/3$ or $3/5$?
Hint: Use a common denominator such as 15.

Your thinking space:

4. A pizza has 12 slices. You eat $1/3$. How many slices remain?
Hint: Find one-third of 12 first.

Your thinking space:

5. What fraction is 35%?
Hint: Write 35/100 and simplify.

Your thinking space:

6. A shop gives 10% off Rs. 500. What is the saving?
Hint: 10% means one tenth.

Your thinking space:

7. Write 0.75 as a fraction and percent.
Hint: 0.75 = 75/100.

Your thinking space:

8. A bottle is $2/5$ full. What percent is empty?
Hint: Full is 1 or 100%.

Your thinking space:

9. What does 1 ppm mean in words?
Hint: One part out of one million equal parts.

Your thinking space:

10. A class of 40 students has 25% absent. How many are absent?
Hint: 25% is one quarter.

Your thinking space:

Classroom practice: Problems 11-20

11. Rani eats $1/3$ of a cake and shares the rest equally between two children. What fraction does each child get?
Hint: First find the remaining fraction.

Your thinking space:

12. A jar has $3/4$ cup jelly. Each sandwich uses $1/5$ cup. How many full sandwiches can be made?
Hint: Divide $3/4$ by $1/5$ and then think about whole sandwiches.

Your thinking space:

13. A team wins 70% of 50 games. How many games did it win and lose?
Hint: 70% = 0.70.

Your thinking space:

14. A solution has 50 ppm chemical in 2 liters. Approximate the amount in milligrams.
Hint: For water, ppm can be treated as mg/L.

Your thinking space:

15. A lake has 8 ppb toxin in 10 million liters. Convert to grams approximately for water.
Hint: 8 ppb is 8 micrograms per liter.

Your thinking space:

16. A recipe uses $3/4$ cup sugar. You make $1/3$ of the recipe. How much sugar is needed?
Hint: Multiply the fractions.

Your thinking space:

17. A shirt price rises by 20% and then falls by 20%. Is the final price the original price?
Hint: Try Rs. 100 as a starting value.

Your thinking space:

18. A class survey: 5% prefer A, 10% prefer B, rest no preference. What fraction has no preference?
Hint: Subtract from 100% and simplify.

Your thinking space:

19. A container has 1000 mL of 15% salt solution. How much pure water must be added to make 10%?
Hint: The salt amount stays the same.

Your thinking space:

20. A room has volume 1000 cubic feet and dust concentration 25 ppm by volume. What fraction of the air volume is dust?
Hint: Write 25/1,000,000 first.

Your thinking space:

Super challenge: Problems 21-30

21. Three friends share a cake. A eats $1/4$, B eats $2/5$ of what remains, and C eats half of the new remainder. What fraction is left?
Hint: Work step by step on the remaining amount, not the original each time.

Your thinking space:

22. A price is discounted 30%, then a tax of 10% is added. For Rs. 1000, find final price and explain why it is not simply 20% off.
Hint: Apply one percent change at a time.

Your thinking space:

23. A tank with 400 ppm mud is diluted by adding clean water until volume doubles. What is the new ppm? Explain using fraction language.
Hint: Mud amount stays constant; total volume doubles.

Your thinking space:

24. A medicine dose is 2 mg per kg of body weight. A 35 kg child has syrup of 50 mg per 5 mL. How many mL are needed?
Hint: Find required mg, then use the concentration ratio.

Your thinking space:

25. A student claims $1/3 + 1/4 = 2/7$. Draw a picture to prove the claim false and give the correct sum.
Hint: Use twelfths.

Your thinking space:

26. A school election has 600 votes. Candidate A gets 42%, B gets 35%, and C gets the

rest. How many more votes does A get than C?
Hint: Find C percent first.

Your thinking space:

27. A 10 L aquarium has nitrate at 20 ppm. A child replaces 2 L with clean water. Estimate new ppm if mixing is perfect.
Hint: Removing 2 L removes the same fraction of nitrate.

Your thinking space:

28. A chocolate bar loses $1/2$ of its remaining mass each day to a magical well. After 6 days, what fraction remains?

Hint: Multiply by 1/2 repeatedly.

Your thinking space:

29. Create a real-life situation where 0.1%, 1% and 10% would feel very different. Use numbers to prove it.
Hint: Choose a base such as 1000 rupees or 1000 people.

Your thinking space:

30. A scientist reports 3 ppb lead in water. A child drinks 2 liters daily for 365 days. Estimate total micrograms of lead.
Hint: 3 ppb in water is about 3 micrograms per liter.

Your thinking space:

Notebook reflection. Which problem forced you to draw a diagram? Which problem had the most surprising idea? Write one mistake you made and how you fixed it.

Chapter 4

Ratios, Rates, Proportion and Mixtures

Classroom question: How do comparisons help us scale recipes, maps, speeds and mixtures?

Learning goals

- Use ratios to compare quantities.
- Scale ratios up and down.
- Solve unit-rate and proportion problems.
- Model mixtures and combined ratios.

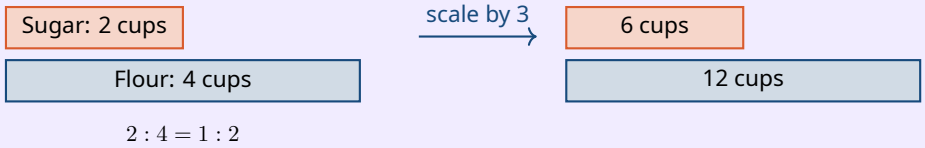
Theory and explanation basics

A ratio compares quantities. It is not always a part of a whole. In a box with 10 red balls and 20 blue balls, the ratio red to blue is 10:20 or 1:2, while the fraction of red balls is 10 out of 30 or 1/3. This distinction prevents many mistakes.

A rate is a ratio with different units, such as kilometers per hour or rupees per kilogram. Proportion means two ratios are equal. Mixture problems become easier when we track the amount of the valuable ingredient: sugar, salt, acid, money or legs in an animal puzzle.

Visual idea

Ratio compares quantities; proportion keeps the comparison unchanged.



$2 : 4 = 1 : 2$

Watch and learn further

- Dr. Pal playlist
- Math Antics - ratios and rates
- Khan Academy - rates and percentages
- Dr. Pal - assignment revision on ratios

Two groups of online learning links

Group A: Dr. Biplab Pal class links

- Full Dr. Pal playlist - ratio and comparison classes
- Dr. Pal - Ratio
- Dr. Pal - Time and distance / age
- Dr. Pal - Inflation percent as rate of change

Group B: Other YouTube references

- Khan Academy YouTube - Ratios, rates and percentages
- Math Antics - Proportions
- YouTube search - unit rates middle school
- YouTube search - ratio tape diagrams

AI prompts for this chapter

Use these prompts after trying the ideas yourself. Ask for hints before full answers.

1. *"Explain the difference between a ratio, a fraction and a rate using mangoes and money."*
2. *"Draw a double number line for this ratio problem and give me a hint: [paste problem]."*
3. *"Create a table to scale this recipe from 4 people to 10 people: [paste recipe]."*
4. *"Ask me questions that help me solve a mixture problem without using a formula first."*
5. *"Check whether my answer has the correct unit. My work is: [paste work]."*

Solved classroom examples

Red and blue balls
10 red and 20 blue gives red:blue = 10:20 = 1:2. Red fraction is $10/(10+20) = 1/3$. Ratio and fraction answer different questions.

Recipe scaling
Sugar:flour = 2:4 = 1:2. If sugar becomes 6 cups, multiply by 3, so flour becomes 12 cups.

Mixture concentration
2 L of 10% salt solution contains 0.2 L salt. 3 L of 20% contains 0.6 L salt. Total salt = 0.8 L in 5 L, so concentration = 16%.

Exercises: 30 problems with hints

Easy practice: Problems 1-10

1. Simplify the ratio 5:15.
Hint: Divide both numbers by 5.

Your thinking space:

2. A box has 10 red and 20 blue balls. Find red:blue.
Hint: Write the two quantities in order.

Your thinking space:

3. A recipe uses 2 cups sugar and 4 cups flour. Simplify sugar:flour.
Hint: Divide by 2.

Your thinking space:

4. If 3 notebooks cost Rs. 60, what is the cost per notebook?
Hint: Divide total cost by number.

Your thinking space:

5. Two numbers are in ratio 3:5. If the first is 9, find the second.
Hint: 3 parts = 9.

Your thinking space:

6. A map scale says 1 cm represents 5 km. What does 3 cm represent?
Hint: Multiply by 3.

Your thinking space:

7. A car travels 120 km in 3 hours. Find speed.
Hint: Distance/time.

Your thinking space:

8. Share 30 candies in ratio 2:3.
Hint: There are 5 total parts.

Your thinking space:

9. Which is faster: 100 km in 2 h or 120 km in 3 h?
Hint: Compare km per hour.

Your thinking space:

10. Write a ratio for boys:girls if there are 12 boys and 18 girls.
Hint: Simplify 12:18.

Your thinking space:

Classroom practice: Problems 11-20

11. A restaurant has male:female workers = 2:3. There are 15 workers. How many of each?
Hint: Total parts = 5.

Your thinking space:

12. A family age ratio father:mother:child = 6:5:2. Father is 48. Find the others.
Hint: One part = 48/6.

Your thinking space:

13. A farm has cows in ratio red:black:white:brown = 3:2:4:1 and total 60. Find each.
Hint: Total parts = 10.

Your thinking space:

14. A company has men:women = 3:2 among 100 workers. If 20 men leave, find the new ratio.
Hint: Find original numbers first.

Your thinking space:

15. A land-to-house area ratio is 4:5. Total is 450 square meters. Find each area.
Hint: There are 9 total parts.

Your thinking space:

16. A car uses speeds 60 and 80 km/h for equal distances. Is average speed 70?
Hint: Average speed = total distance/total time.

Your thinking space:

17. A juice mix has orange:water = 3:7. How much orange concentrate is needed for 2 liters?
Hint: 3 out of 10 parts is orange.

Your thinking space:

18. Two oil mixtures are 10% and 30%. How much of each is needed to make 10 L of 18% mixture?
Hint: Let one amount be x; track pure oil.

Your thinking space:

19. The Alien Zoo has ratios T:Q = 2:3 and Q:P = 4:5. Find T:Q:P.
Hint: Make Q the same using common multiples.

Your thinking space:

20. A cyclist covers one part of a route at 12 km/h and another equal time at 18 km/h. Find average speed.
Hint: Equal times make average speed arithmetic mean.

Your thinking space:

Super challenge: Problems 21-30

21. A shop mixes rice costing Rs. 30/kg and Rs. 50/kg to make 20 kg costing Rs. 42/kg. How many kg of each rice?
Hint: Track total cost as well as total mass.

Your thinking space:

22. A map is accidentally printed 20% larger. If the printed scale reads 1 cm = 4 km,

what is the true scale?
Hint: A printed centimeter represents less original map length.

Your thinking space:

23. A painter mixes blue and yellow in ratio 5:3. After adding 4 cups yellow, the ratio becomes 5:4. How much blue was there?
Hint: Let original common multiplier be k.

Your thinking space:

24. A tank has 12 L of 25% acid. How much water must be added to make it 15%?
Hint: Acid amount remains constant.

Your thinking space:

25. A school has boys:girls = 7:8. After 15 boys join, ratio becomes 4:4? Decide whether this is possible and correct the ratio statement if needed.
Hint: 4:4 means 1:1; set up equation.

Your thinking space:

26. Two trains cover the same route. Train A takes 4 h, Train B takes 5 h. Their speeds differ by 15 km/h. Find the route length.
Hint: Let distance be d and compare d/4 and d/5.

Your thinking space:

27. Design a recipe for 30 cookies using a 2:3:5 ratio of sugar:butter:flour and total dry mixture 500 g. Explain assumptions.
Hint: Total parts = 10; decide which ingredients count as dry.

Your thinking space:

28. A rectangle has length:width = 5:3 and perimeter 64 cm. Find area.
Hint: Let sides be 5x and 3x.

Your thinking space:

29. In a class, the ratio of students who like math to science is 4:5, science to art is 3:2. Find a possible smallest group that satisfies both.
Hint: Make science parts equal.

Your thinking space:

30. A mixture problem has two unknowns but only one equation. Explain why infinitely many answers may be possible and give two examples.
Hint: Choose a total amount or a concentration as the missing condition.

Your thinking space:

Notebook reflection. Which problem forced you to draw a diagram? Which problem had the most surprising idea? Write one mistake you made and how you fixed it.

Chapter 5

Pi, Circles, Approximation and Measurement

Classroom question: Why does every circle hide the same mysterious ratio?

Learning goals

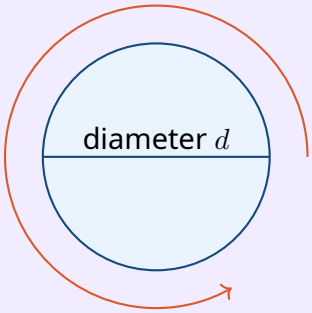
- Understand radius, diameter, circumference and area.
- Use π as a ratio and approximation.
- Estimate by bounding and measuring.
- Solve perimeter and area problems with units.

Theory and explanation basics

A circle is the set of all points the same distance from a center. That distance is the radius. The diameter is twice the radius. The circumference is the distance around the circle. The ratio circumference divided by diameter is always the same number, called π.

Approximation is not guessing randomly. Archimedes estimated π by trapping a circle between polygons inside and outside it. Modern students can do the same idea by measuring, bounding and checking reasonableness. In geometry, a drawing is not decoration; it is a thinking tool.

Visual idea



One full trip around the circle is the circumference C.

$\pi = \frac{C}{d}$, so $C = \pi d = 2\pi r$.

Watch and learn further

- Dr. Pal playlist
- Dr. Pal - PI topic in class materials
- Math Antics - geometry
- Khan Academy - geometry

Two groups of online learning links

Group A: Dr. Biplab Pal class links

- Full Dr. Pal playlist - geometry and pi path
- Dr. Pal - Magic of Pi
- Dr. Pal - Geometry
- Dr. Pal - Basic Geometry

Group B: Other YouTube references

- Numberphile - Pi
- Numberphile - Pi playlist
- YouTube search - Khan Academy circumference and area of circles
- YouTube search - Math Antics circles

AI prompts for this chapter

Use these prompts after trying the ideas yourself. Ask for hints before full answers.

1. *"Explain pi using string wrapped around three circular objects at home."*
2. *"Show me why circumference divided by diameter is nearly the same for all circles using a simple table."*
3. *"Give me a hint for a wheel rotation problem, but do not solve it fully: [paste problem]."*
4. *"Make a diagram-based explanation for area of a circle for a 10-year-old."*
5. *"Check if I used radius or diameter correctly in this solution: [paste work]."*

Solved classroom examples

Circumference from diameter
A wheel has diameter 70 cm. $C = \pi d \approx 3.14 \times 70 = 219.8$ cm. One rotation moves the bicycle about 2.2 m.

Area from radius
A circular garden has radius 4 m. Area $= \pi r^2 \approx 3.14 \times 16 = 50.24$ square meters.

Bounding idea
If a circle is inside a square of side 10 cm, the circle area is less than 100 square cm. If it contains a smaller square, the area is more than that smaller square. Bounds help check answers.

Exercises: 30 problems with hints

Easy practice: Problems 1-10

1. Name the radius and diameter of a circle if radius is 5 cm.
Hint: Diameter is twice the radius.

Your thinking space:

2. Find circumference of a circle with diameter 10 cm using $\pi \approx 3.14$.
Hint: Use C = pi times d.

Your thinking space:

3. Find area of a circle with radius 3 cm.
Hint: Use A = pi r squared.

Your thinking space:

4. A wheel radius is 7 cm. What is its diameter?
Hint: Double the radius.

Your thinking space:

5. Which unit fits area: cm or square cm?
Hint: Area measures covering.

Your thinking space:

6. A square has side 6 cm. Find its perimeter and area.
Hint: Perimeter adds edges; area multiplies side by side.

Your thinking space:

7. A circle has circumference about 31.4 cm. Estimate its diameter.
Hint: Divide by 3.14.

Your thinking space:

8. Draw a circle and label center, radius and diameter.
Hint: The diameter passes through the center.

Your thinking space:

9. If a circular pizza is cut into 8 equal slices, what fraction is 3 slices?
Hint: Numerator is selected slices.

Your thinking space:

10. Why is π not exactly 3?
Hint: Measurement shows circumference is slightly more than 3 diameters.

Your thinking space:

Classroom practice: Problems 11-20

11. A bicycle wheel has diameter 60 cm. How far does it travel in 20 rotations?
Hint: Find one circumference, then multiply by 20.

Your thinking space:

12. A circular pond has radius 5 m. A fence is built around it. Estimate fence length.
Hint: Use circumference.

Your thinking space:

13. A circular rug has diameter 2.8 m. Estimate the area covered.
Hint: Radius is half the diameter.

Your thinking space:

14. A race track has two straight 100 m sections and two semicircles of radius 20 m. Find one lap length.
Hint: Two semicircles make one full circle.

Your thinking space:

15. A pizza of radius 14 cm is shared equally by 4 friends. Estimate area per friend.
Hint: Find total area and divide by 4.

Your thinking space:

16. A square is drawn around a circle of radius 5 cm. Find square area and compare with circle area.
Hint: The square side equals the diameter.

Your thinking space:

17. A circular table has radius 0.75 m. How much lace is needed around the edge?
Hint: Use circumference with meters.

Your thinking space:

18. A clock hand is 10 cm long. How far does its tip move in one full rotation?
Hint: The hand is the radius.

Your thinking space:

19. If a circle has area 78.5 square cm, estimate its radius.
Hint: Solve pi r squared = 78.5.

Your thinking space:

20. A student uses diameter instead of radius in the area formula. Show the error for diameter 10 cm.
Hint: Compare pi times 10 squared with pi times 5 squared.

Your thinking space:

Super challenge: Problems 21-30

21. Archimedes used polygons to bound π. Explain how a hexagon inside a circle and a hexagon outside a circle give lower and upper estimates.
Hint: Compare perimeters with circumference.

Your thinking space:

22. A goat is tied to a corner of a 10 m by 8 m shed with a 6 m rope. What grazing area is reachable if the shed blocks movement?

Hint: Draw a quarter circle.

Your thinking space:

23. A circular path of width 1 m surrounds a pond of radius 6 m. Find the area of the path.
Hint: Subtract inner circle area from outer circle area.

Your thinking space:

24. A wheel of radius 35 cm makes 1000 rotations. Estimate distance in kilometers.
Hint: Convert centimeters to meters and kilometers.

Your thinking space:

25. A square and a circle have the same perimeter 40 cm. Which has greater area?
Hint: Find each area and compare.

Your thinking space:

26. Design a method to estimate π using string and three circular objects at home. Include a table.
Hint: Measure circumference and diameter for each.

Your thinking space:

27. A circular garden is divided into a 90-degree sector and the rest. If radius is 8 m, find area of each part.
Hint: A 90-degree sector is one quarter.

Your thinking space:

28. A circular logo must fit inside a rectangular page 18 cm by 12 cm. What is the largest possible radius?
Hint: The limiting dimension is the smaller side.

Your thinking space:

29. A pizza place offers 10-inch and 12-inch pizzas. The 12-inch costs 40% more. Which is better value by area?
Hint: Area depends on radius squared, not diameter directly.

Your thinking space:

30. A track is shown as a rectangle 50 m by 30 m with semicircles on the short sides. Find perimeter and enclosed area.
Hint: Two semicircles make one circle of radius 15 m.

Your thinking space:

Notebook reflection. Which problem forced you to draw a diagram? Which problem had the most surprising idea? Write one mistake you made and how you fixed it.

Chapter 6

Equations: Balance, Remainders and One Unknown

Classroom question: How can a single unknown represent an entire story?

Learning goals

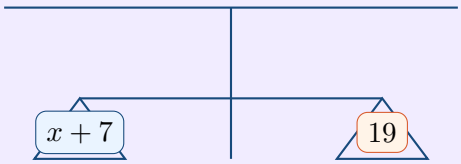
- Solve linear equations in one variable.
- Use balance operations correctly.
- Translate age, money, speed and remainder problems.
- Check answers in the original story.

Theory and explanation basics

An equation says two expressions have the same value. Solving is not magic; it is keeping balance while making the unknown visible. If $x + 7 = 19$, subtract 7 from both sides. If $3x = 24$, divide both sides by 3. The operation must be fair to both sides.

Harder problems hide the equation inside a story. In speed problems, distance = speed times time. In work problems, rate means part of the job per unit time. In remainder problems, a number that leaves remainder 4 when divided by 7 can be written as $7k + 4$.

Visual idea



An equation is a balance.
Whatever you do to one side,
do the same to the other.

Watch and learn further

- Dr. Pal playlist
- Dr. Pal - Equation 1
- Dr. Pal - Basics of Equation
- Khan Academy - algebra basics

Two groups of online learning links

Group A: Dr. Biplab Pal class links

- Full Dr. Pal playlist - equations and algebra basics
- Dr. Pal - Equation 1
- Dr. Pal - What is equation?
- Dr. Pal - Problem Solving with one variable

Group B: Other YouTube references

- YouTube search - Khan Academy algebra basics equations
- YouTube search - Math Antics algebra basics
- YouTube search - balance model equations
- YouTube search - one step and two step equations

AI prompts for this chapter

Use these prompts after trying the ideas yourself. Ask for hints before full answers.

1. *"Explain an equation as a balance scale using objects on both sides."*
2. *"For this equation, give me one hint and one drawing idea, not the answer: [paste equation]."*
3. *"Turn this word problem into an equation, then stop before solving: [paste problem]."*
4. *"Ask me to solve this equation step by step, checking each line: [paste equation]."*
5. *"Find the first algebra mistake in my solution but do not finish the problem: [paste work]."*

Solved classroom examples

Basic balance
Solve $x + 7 = 19$. Subtract 7 from both sides: $x = 12$. Check: $12 + 7 = 19$.

Age equation
Five times the age 5 years ago subtracted from three times the age 5 years from now gives the present age: $3(x + 5) - 5(x - 5) = x$. Then $3x + 15 - 5x + 25 = x$, so $40 - 2x = x$, $x = 40/3$. Since age is fractional, discuss whether the story is realistic.

Round trip
Distance each way is d. Time going: $d/50$, return: $d/30$. If total is 7 h, $d/50 + d/30 = 7$. Then $8d/150 = 7$, so $d = 131.25$ miles.

Exercises: 30 problems with hints

Easy practice: Problems 1-10

1. Solve $x + 5 = 12$.
Hint: Undo +5 by subtracting 5.

Your thinking space:

2. Solve $3x = 21$.
Hint: Divide by 3.

Your thinking space:

3. Solve $x/4 = 8$.
Hint: Multiply by 4.

Your thinking space:

4. Solve $2x + 3 = 15$.
Hint: Subtract 3, then divide by 2.

Your thinking space:

5. A number plus 9 is 20. Find the number.
Hint: Translate directly.

Your thinking space:

6. A pencil costs Rs. x. Five pencils cost Rs. 60. Find x.
Hint: Set 5x=60.

Your thinking space:

7. Check whether x=4 solves $3x + 2 = 14$.
Hint: Substitute 4.

Your thinking space:

8. Solve $10 - x = 6$.
Hint: Think: what must be subtracted from 10 to get 6?

Your thinking space:

9. Write an equation for: twice a number is 18.
Hint: Use 2x=18.

Your thinking space:

10. Solve $5x - 5 = 20$.
Hint: Add 5, then divide by 5.

Your thinking space:

Classroom practice: Problems 11-20

11. A worker earns Rs. 50 per working day and loses Rs. 10 per absent day. In 30 days he earns Rs. 1200. How many days did he work?
Hint: Let worked days be x and absent days 30-x.

Your thinking space:

12. A shopkeeper sells half his items at 20% profit and half at 10% loss. If total cost is Rs. 2000, find total profit.
Hint: This is a check problem; compute each half separately.

Your thinking space:

13. A cyclist travels to a town at 15 km/h and returns at 10 km/h. Total time is 5 h. Find distance one way.
Hint: Use d/15 + d/10 = 5.

Your thinking space:

14. The cost of 5 apples and 3 oranges is Rs. 230. One apple costs twice one orange.

Find prices.
Hint: Let orange price be x.

Your thinking space:

15. A man borrows Rs. 1500 at 8% simple interest. After how many years will amount be Rs. 1800?
Hint: Interest = principal x rate x time.

Your thinking space:

16. A number divided by 5 leaves remainder 3 and divided by 4 leaves remainder 2. Find the smallest positive number.
Hint: List numbers 3 more than multiples of 5.

Your thinking space:

17. A book is split into two parts; first has 40 more pages than second; total is 300. Find pages.
Hint: Let second part be x.

Your thinking space:

18. A theater has 25 rows. Adding 10 seats to each row adds how many seats? Reverse the question for 300 extra seats.
Hint: Extra seats = rows times extra per row.

Your thinking space:

19. A mixture of Rs. 3/kg rice and Rs. 5/kg rice costs Rs. 4/kg. What can you say about amounts?
Hint: Equal cost means likely equal masses; prove with equation.

Your thinking space:

20. A number between 1 and 21 satisfies $N = 7y + 4 = 3x + 2$. Find it.
Hint: Try y=0,1,2.

Your thinking space:

Super challenge: Problems 21-30

21. A car covers a distance at 60 km/h and returns at 80 km/h. The total time is 7 h. Find one-way distance.
Hint: Use d/60 + d/80 = 7.

Your thinking space:

22. Two workers together complete a job in 6 h. One alone takes 10 h. How long does the other take?
Hint: Use rates: 1/6 = 1/10 + 1/x.

Your thinking space:

23. A water tank is filled by one pipe in 8 h and emptied by another in 12 h. If both are open, how long to fill?
Hint: Net rate = filling rate - emptying rate.

Your thinking space:

24. A student solves $3(x-2)=2x+5$ and gets x=11. Check and correct.
Hint: Expand first, then solve.

Your thinking space:

25. A two-digit number has digit sum 12. Reversing digits increases the number by 18. Find the number.
Hint: Let tens digit be a and ones digit be b.

Your thinking space:

26. A train 120 m long passes a pole in 6 s. How long to pass a 180 m bridge at same speed?
Hint: Find speed first; total distance is train plus bridge.

Your thinking space:

27. A product price after 15% discount is Rs. 850. Find original price.
Hint: 0.85 times original = 850.

Your thinking space:

28. A number leaves remainder 1 when divided by 2, 3 and 4, but is divisible by 5. Find the smallest such number.

Hint: Numbers one more than multiples of lcm(2,3,4).

Your thinking space:

29. Create a problem that leads to $x/3 + x/4 = 14$. Solve it and explain the units.
Hint: Use sharing or time spent.

Your thinking space:

30. An equation has no solution: $2x + 5 = 2x + 9$. Explain using the balance idea and create a story for it.
Hint: Subtract 2x from both sides.

Your thinking space:

Notebook reflection. Which problem forced you to draw a diagram? Which problem had the most surprising idea? Write one mistake you made and how you fixed it.

Chapter 7

Geometry: Shapes, Angles, Triangles and Proof Thinking

Classroom question: How can diagrams become a language for reasoning?

Learning goals

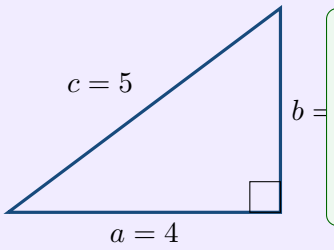
- Recognize common shapes and their properties.
- Use angle facts with triangles and parallel lines.
- Apply Pythagoras in right triangles.
- Use similarity and scale in indirect measurement.

Theory and explanation basics

Geometry begins with simple objects: point, line, angle, triangle, rectangle and circle. But its real power is proof. A diagram suggests relationships; reasoning confirms them. In a triangle, the angles add to 180 degrees. In parallel lines cut by a transversal, many equal angles appear.

A right triangle has one 90-degree angle. The Pythagorean theorem says the square on the longest side equals the sum of the squares on the two shorter sides. Similar triangles have the same shape but different sizes; their matching sides keep the same ratio.

Visual idea



Right triangle idea: the square on the longest side equals the sum of the two smaller squares.

$$a^2 + b^2 = c^2$$

Watch and learn further

- Dr. Pal playlist
- Dr. Pal - Basic Geometry
- Dr. Pal - Triangle and other shapes
- Dr. Pal - Geometry and Algebra

Two groups of online learning links

Group A: Dr. Biplab Pal class links

- Full Dr. Pal playlist - geometry path
- Dr. Pal - Basic Geometry
- Dr. Pal - Triangle and other shapes
- Dr. Pal - Geometry 3

Group B: Other YouTube references

- Math Antics YouTube playlists - Geometry sections
- YouTube search - Khan Academy triangles angles
- YouTube search - geometry proof thinking middle school
- YouTube search - area perimeter visual explanation

AI prompts for this chapter

Use these prompts after trying the ideas yourself. Ask for hints before full answers.

1. *"Explain triangle angle sum using a paper-tearing activity."*
2. *"Make a diagram for this geometry problem and label every given length or angle: [paste problem]."*
3. *"Give me only a hint about which theorem or idea to use here: [paste problem]."*
4. *"Ask me to justify each step of my geometry solution like a mini-proof."*
5. *"Create a real-life example where area and perimeter behave differently."*

Solved classroom examples

Triangle angles
If two angles of a triangle are 55 degrees and 65 degrees, the third is $180-55-65=60$ degrees.

Pythagoras
In a right triangle with legs 6 and 8, hypotenuse c satisfies $6^2+8^2=c^2$. So $36+64=100$, and $c=10$.

Scale by shadows
A 1.5 m stick casts a 2 m shadow. A tree casts an 8 m shadow. Similar triangles give $1.5/2=h/8$, so $h=6$ m.

Exercises: 30 problems with hints

Easy practice: Problems 1-10

1. Name a real-life object shaped like a rectangle.
Hint: A door or book page works.

Your thinking space:

2. How many degrees are in a right angle?
Hint: A right angle is a square corner.

Your thinking space:

3. Find the third angle of a triangle with angles 70 and 50 degrees.
Hint: Triangle sum is 180 degrees.

Your thinking space:

4. Find perimeter of a rectangle 8 cm by 5 cm.
Hint: Add all four sides.

Your thinking space:

5. Find area of a rectangle 8 cm by 5 cm.
Hint: Multiply length by width.

Your thinking space:

6. Is a square also a rectangle? Explain.
Hint: Check the definition of rectangle.

Your thinking space:

7. Find the hypotenuse of a right triangle with legs 3 and 4.
Hint: Use the 3-4-5 triangle.

Your thinking space:

8. A triangle has sides 2, 3 and 10. Is it possible?
Hint: Use triangle inequality.

Your thinking space:

9. Draw an acute angle and an obtuse angle.
Hint: Acute is less than 90; obtuse is more than 90.

Your thinking space:

10. A circle and square both have centers. How are those centers different?
Hint: Circle center is equidistant from boundary.

Your thinking space:

Classroom practice: Problems 11-20

11. Two parallel lines are cut by a transversal. If one angle is 65 degrees, find all angle sizes in the diagram.
Hint: Use vertical, corresponding and supplementary angles.

Your thinking space:

12. A ladder 10 m long reaches a wall. Its foot is 6 m from the wall. How high does it reach?
Hint: Right triangle with hypotenuse 10.

Your thinking space:

13. A rectangular field 30 m by 40 m has a diagonal path. Find the path length.
Hint: Use Pythagoras.

Your thinking space:

14. A polygon has 8 sides. Find the sum of its interior angles.
Hint: Use (n-2) times 180 degrees.

Your thinking space:

15. A parallelogram has one angle 70 degrees. Find the other angles.
Hint: Opposite equal; adjacent supplementary.

Your thinking space:

16. Two similar triangles have side ratio 2:5. A small side of 6 cm matches a large side. Find it.
Hint: Multiply by 5/2.

Your thinking space:

17. A rhombus has diagonals 10 cm and 24 cm. Find side length.
Hint: Diagonals bisect at right angles.

Your thinking space:

18. A square of side 10 cm contains a diagonal. Find diagonal length exactly and approximately.
Hint: Use $10^2 + 10^2 = d^2$.

Your thinking space:

19. A triangle has sides 5, 5 and 8. Find the height to the base 8.
Hint: Split into two right triangles.

Your thinking space:

20. A point is 3 units from the x-axis and 4 units from the y-axis in the first quadrant. Find its distance from the origin.
Hint: Coordinates are (4,3) or (3,4) depending on axes; draw.

Your thinking space:

Super challenge: Problems 21-30

21. Prove visually that the angles in a triangle add to 180 degrees by cutting and rearranging the corners.
Hint: Place the three angles along a straight line.

Your thinking space:

22. A 13 m ladder leans against a wall, and the foot is 5 m from the wall. If the foot

slides 2 m farther, how far down does the top move?
Hint: Compute heights before and after.

Your thinking space:

23. A triangular field has sides 13, 14 and 15 m. Find area by dropping a height or using Heron's formula if known.
Hint: Try splitting into two right triangles with base 14.

Your thinking space:

24. A rectangle and a square have the same area 144 square cm. The rectangle length is 18 cm. Compare perimeters.

Hint: Find rectangle width and square side.

Your thinking space:

25. Use similar triangles to estimate the height of a building using a 1 m ruler shadow. Create realistic numbers and solve.
Hint: Keep ratios height/shadow equal.

Your thinking space:

26. A circle is tangent to a line at point T. Explain why radius to T is perpendicular to the tangent.
Hint: If not, a shorter distance would reach the line.

Your thinking space:

27. A regular hexagon can be split into six equilateral triangles. If side is 4 cm, find perimeter and area idea.
Hint: Use six triangles of side 4.

Your thinking space:

28. A kite shape has perpendicular diagonals 12 cm and 16 cm. Find area and compare to a rectangle with same diagonal lengths.
Hint: Kite area = half product of diagonals.

Your thinking space:

29. A tiled floor uses right triangles with legs 9 cm and 12 cm. How many tile edges of length 15 cm are in 40 tiles?
Hint: Find hypotenuse first; interpret tile arrangement assumptions.

Your thinking space:

30. A student believes all shapes with four equal sides are squares. Give a counterexample and explain with angle information.
Hint: A rhombus need not have right angles.

Your thinking space:

Notebook reflection. Which problem forced you to draw a diagram? Which problem had the most surprising idea? Write one mistake you made and how you fixed it.

Chapter 8

Two Variables, Graphs, Patterns and Systems

Classroom question: How can a picture of points reveal a relationship?

Learning goals

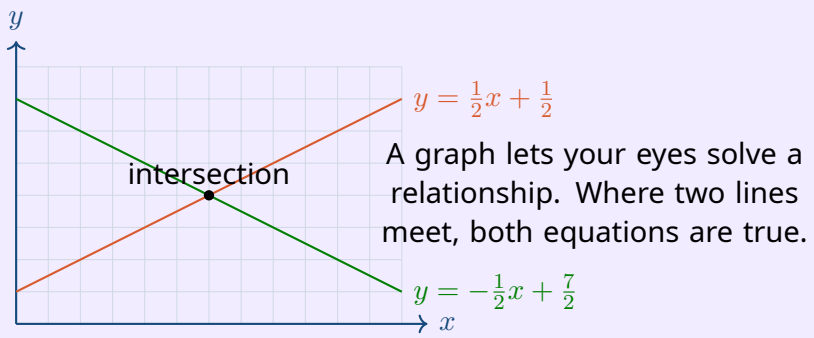
- Use ordered pairs and coordinate axes.
- Read tables, graphs and simple equations.
- Understand slope as change in y per change in x.
- Solve simple systems by graph or substitution.

Theory and explanation basics

When two quantities change together, a table and a graph help us see the pattern. An ordered pair (x, y) tells how far to move horizontally and vertically. A line often represents a constant rate of change. The slope is the amount y changes when x increases by 1.

A system of equations asks for values that satisfy two relationships at once. On a graph, the solution is the intersection point. In a table, it is the row where both rules give the same result. In real life, systems model price combinations, tickets, mixtures and animal-leg puzzles.

Visual idea



Watch and learn further

- Dr. Pal playlist
- Dr. Pal - Equation/two variable class in playlist
- Khan Academy - graphing lines
- 3Blue1Brown - visual math channel

Two groups of online learning links

Group A: Dr. Biplab Pal class links

- Full Dr. Pal playlist - equations, graphs and patterns
- Dr. Pal - Word Problem and Algebra
- Dr. Pal - Equation 1
- Dr. Pal - How to use Math?

Group B: Other YouTube references

- YouTube search - Khan Academy graphing linear equations
- 3Blue1Brown YouTube channel - visual math
- YouTube search - coordinate plane middle school
- YouTube search - systems of equations intro

AI prompts for this chapter

Use these prompts after trying the ideas yourself. Ask for hints before full answers.

1. *"Explain x and y as input and output using a snack shop price table."*
2. *"Make a table and graph for this rule, then ask me what pattern I see: [paste rule]."*
3. *"Give me a hint for solving this pair of equations by reasoning, not by memorized formula: [paste equations]."*
4. *"Create a coordinate-plane story problem that a 12-year-old could draw."*
5. *"Check if my graph matches my equation. My points are [paste points] and equation is [paste equation]."*

Solved classroom examples

Table to rule
If x values 1,2,3 give y values 3,5,7, y increases by 2 each time. A rule is $y = 2x + 1$.

Intersection
For $x + y = 7$ and $800x + 300y = 4100$, use $y = 7 - x$. Then $800x + 300(7 - x) = 4100$, so $500x = 2000$, $x = 4$, $y = 3$.

Slope meaning
If a taxi costs Rs. 50 to start and Rs. 12 per km, cost is $C = 50 + 12d$. The slope 12 means each extra km adds Rs. 12.

Exercises: 30 problems with hints

Easy practice: Problems 1-10

1. Plot the point (3,2) on coordinate axes.
Hint: Move 3 right and 2 up.

Your thinking space:

2. For $y = 2x$, find y when $x = 5$.
Hint: Substitute x=5.

Your thinking space:

3. Complete the table for $y = x + 4$ when x is 0, 1, 2.
Hint: Add 4 to each x.

Your thinking space:

4. Which point lies on $y = x + 1$: (2,3) or (2,4)?
Hint: Check by substitution.

Your thinking space:

5. What is the slope of a line that rises 6 when x increases by 3?
Hint: Slope = rise/run.

Your thinking space:

6. A rule adds 3 each step starting at 5. Write first five terms.
Hint: 5, then add 3 repeatedly.

Your thinking space:

7. If x+y=10 and x=6, find y.
Hint: Substitute x.

Your thinking space:

8. A graph line crosses y-axis at 4. What does that mean in a cost story?
Hint: It is the starting cost when distance is 0.

Your thinking space:

9. Make a table for y = 3x for x = 1 to 5.
Hint: Multiply each x by 3.

Your thinking space:

10. Does (1,5) satisfy 2x+y=7?
Hint: Plug in x=1, y=5.

Your thinking space:

Classroom practice: Problems 11-20

11. A shop sells apples at Rs. 20 and bananas at Rs. 5. You buy 10 fruits for Rs. 95. Find how many apples.
Hint: Use a+b=10 and 20a+5b=95.

Your thinking space:

12. A line passes through (0,3) and (4,11). Find its slope and rule.
Hint: Slope = change in y/change in x.

Your thinking space:

13. Two plans: Plan A costs Rs. 100 plus Rs. 10/day; Plan B costs Rs. 40 plus Rs. 20/day. When are costs equal?
Hint: Set 100+10d = 40+20d.

Your thinking space:

14. The pattern 2, 5, 8, 11 continues. Write the 20th term.
Hint: Find rule using starting value and common difference.

Your thinking space:

15. A rectangle has perimeter 30 and length 2 more than width. Find both using equations.
Hint: Let width be w.

Your thinking space:

16. Solve by substitution: x+y=9 and x-y=3.
Hint: Add or substitute.

Your thinking space:

17. A graph shows distance increasing 15 km every hour. What is the slope and meaning?
Hint: Slope is speed.

Your thinking space:

18. A class sells adult tickets Rs. 50 and child tickets Rs. 20. It sells 30 tickets for Rs. 960. Find each type.
Hint: Use total tickets and total money.

Your thinking space:

19. A sequence doubles each time starting at 3. Write first six terms and compare with adding 3 each time.
Hint: Multiplication growth gets faster.

Your thinking space:

20. Draw two lines that never meet. What is true about their slopes?
Hint: Parallel lines have same slope.

Your thinking space:

Super challenge: Problems 21-30

21. Two candles burn at different rates. Candle A starts at 20 cm and loses 2 cm/h. Candle B starts at 12 cm and loses 1 cm/h. When are they equal height?
Hint: Set 20-2t = 12-t.

Your thinking space:

22. A phone plan has a fixed fee and per-GB fee. 3 GB costs Rs. 290 and 8 GB costs Rs. 540. Find the fixed fee and per-GB fee.
Hint: Let cost = fixed + rate times GB.

Your thinking space:

23. A point moves by rule: each step x increases by 2 and y increases by 3. Starting at (1,1), find the 10th point.
Hint: Count nine moves after the first point.

Your thinking space:

24. A system has no solution. Create a real-life story for two parallel cost lines.

Hint: Use same per-unit rate but different starting fees.

Your thinking space:

25. A zoo has chickens and goats, 18 animals and 50 legs. Solve using a graph or equations.
Hint: c+g=18, 2c+4g=50.

Your thinking space:

26. A student draws a graph with time on y-axis and distance on x-axis. How does the meaning of slope change?
Hint: Slope becomes time per distance, not speed.

Your thinking space:

27. A pattern alternates: 2, 5, 4, 10, 8, 20,... Describe two possible rules and predict the next terms.
Hint: Look at odd and even positions separately.

Your thinking space:

28. A rectangle area is 84 and perimeter is 38. Find integer side lengths.
Hint: Let length and width; use factor pairs.

Your thinking space:

29. Two trains leave cities 300 km apart and travel toward each other at 40 and 60 km/h. Make a graph of distance between trains over time and find meeting time.
Hint: Distance decreases by combined speed.

Your thinking space:

30. An online game score is $S = 50 + 20w - 5l$. Find combinations of wins and losses that give 165. Are all combinations possible?
Hint: Solve 20w-5l=115 and consider whole numbers.

Your thinking space:

Notebook reflection. Which problem forced you to draw a diagram? Which problem had the most surprising idea? Write one mistake you made and how you fixed it.

Chapter 9

Statistics, Data, Spreadsheets and Fair Comparisons

Classroom question: How do numbers become evidence instead of noise?

Learning goals

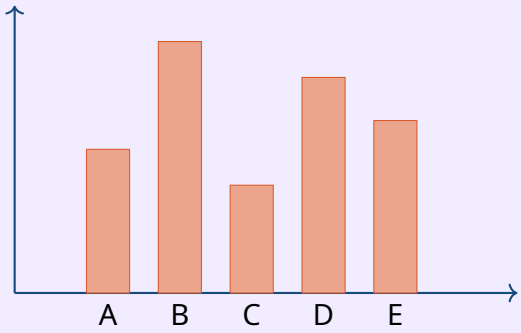
- Compute mean, median, mode and range.
- Read bar charts, line graphs and tables.
- Identify bias and misleading comparisons.
- Use spreadsheet-style thinking for repeated calculations.

Theory and explanation basics

Statistics begins when we collect data. But data is not automatically truth. We must ask what was measured, how the sample was chosen and whether the comparison is fair. Averages summarize data, but different averages answer different questions.

Mean is the balance point: add values and divide by the count. Median is the middle after sorting. Mode is the most common value. Range is largest minus smallest. Graphs help us see patterns, but graphs can also mislead if axes are cut or scales are uneven.

Visual idea



Data becomes meaningful only after we ask: What was measured? How was it collected? What comparison is fair?

Watch and learn further

- Dr. Pal playlist
- Dr. Pal - Understanding Data
- Khan Academy - statistics and probability
- Math Antics - statistics and measurement

Two groups of online learning links

Group A: Dr. Biplab Pal class links

- Full Dr. Pal playlist - data thinking and applications
- Dr. Pal - Understanding Data
- Dr. Pal - Sense of reference
- Dr. Pal - How to use Math?

Group B: Other YouTube references

- YouTube search - Khan Academy statistics middle school
- YouTube search - Math Antics mean median mode
- YouTube search - misleading graphs for students
- YouTube search - spreadsheets for kids math

AI prompts for this chapter

Use these prompts after trying the ideas yourself. Ask for hints before full answers.

1. *"Explain mean, median and mode using test scores with one very large outlier."*
2. *"Create a small data table and ask me which graph is best: bar, line or pie chart."*
3. *"Give me a hint for spotting whether this graph is misleading: [describe graph]."*
4. *"Turn this classroom survey into percentages and a fair comparison: [paste data]."*
5. *"Check my spreadsheet formula idea for this problem: [paste formula and goal]."*

Solved classroom examples

Mean and median
Data: 4, 5, 6, 7, 100. Mean is $122/5 = 24.4$, but median is 6. The outlier 100 pulls the mean upward.

Bar chart reading
If A has 20 votes, B has 35 and C has 15, total votes are 70. Bś percent is $35/70 = 50\%$.

Spreadsheet formula
If prices are in cells A2:A11, a spreadsheet can compute total using SUM and average using AVERAGE. The idea is repeated arithmetic with labels.

Exercises: 30 problems with hints

Easy practice: Problems 1-10

1. Find the mean of 2, 4, 6, 8.
Hint: Add and divide by 4.

Your thinking space:

2. Find the median of 3, 9, 5, 1, 7.
Hint: Sort first.

Your thinking space:

3. Find the mode of 2, 3, 3, 4, 5.
Hint: Look for most frequent value.

Your thinking space:

4. Find the range of 12, 7, 20, 10.
Hint: Largest minus smallest.

Your thinking space:

5. A bar chart shows 10 cats and 15 dogs. How many animals total?
Hint: Add categories.

Your thinking space:

6. Convert 18 out of 60 votes to a percent.
Hint: 18/60 times 100.

Your thinking space:

7. Which is better for typical value when one number is extreme: mean or median?
Hint: Think about outliers.

Your thinking space:

8. Make a tally table for favorite fruits in your family.
Hint: Use one tally per person.

Your thinking space:

9. What question should you ask before trusting survey data?
Hint: Ask who was surveyed and how many.

Your thinking space:

10. A line graph goes up from Monday to Tuesday. What does that mean?
Hint: It means the measured value increased.

Your thinking space:

Classroom practice: Problems 11-20

11. Test scores are 60, 70, 80, 90, 100. Find mean, median and range.
Hint: Sort, add and compare.

Your thinking space:

12. Class A has mean 80 with 10 students. Class B has mean 70 with 30 students. Find combined mean.
Hint: Use total score, not average of averages.

Your thinking space:

13. A graph starts its vertical axis at 90 instead of 0. Explain how this can exaggerate change.
Hint: Small differences look tall.

Your thinking space:

14. A store records sales: Mon 12, Tue 18, Wed 15, Thu 25, Fri 30. Find mean daily

sales.
Hint: Total divided by 5.

Your thinking space:

15. A survey of 20 students says 70% like football. How many students is that?
Hint: 70% of 20.

Your thinking space:

16. Create a dot plot for data 1,2,2,3,3,3,4.
Hint: Stack dots above each number.

Your thinking space:

17. Find the missing number if the mean of 6, 8, x, 10 is 9.
Hint: Total must be 36.

Your thinking space:

18. A dataset has median 12 and range 10. Give one possible dataset of five numbers.
Hint: Build sorted numbers with middle 12.

Your thinking space:

19. Two children compare pocket money: one uses weekly amount, one uses monthly amount. Why is comparison unfair?
Hint: Units differ.

Your thinking space:

20. A spreadsheet column has price and quantity. Write a formula idea for total cost.
Hint: Multiply price by quantity in each row, then sum.

Your thinking space:

Super challenge: Problems 21-30

21. A class has scores with mean 75. One student with score 0 was absent due to illness. Should the 0 be included? Give two arguments.
Hint: Statistics involves context, not only calculation.

Your thinking space:

22. A cricket player scores 0, 0, 100, 100. Another scores 45, 50, 55, 50. Compare mean and consistency.
Hint: Compute mean and range.

Your thinking space:

23. A town says pollution fell by 50%, from 2 ppm to 1 ppm. Another town fell from 100 ppm to 70 ppm. Which improvement is more important?
Hint: Compare percent and absolute change.

Your thinking space:

24. Design a fair survey to learn whether students like math videos. Include sample

size, question wording and possible bias.
Hint: Avoid asking only students who already watch videos.

Your thinking space:

25. A bar chart has unequal widths for bars. Explain why this may mislead and redraw the idea fairly.
Hint: Bar height should represent value; width should not imply more unless intentional.

Your thinking space:

26. A grocery spreadsheet has 100 rows. Explain how formulas reduce errors compared with hand calculation.

Hint: Formulas repeat the same rule consistently.

Your thinking space:

27. A dataset is 1, 1, 2, 2, 100. Which average would you report for typical family size and why?
Hint: Think about meaning of 100.

Your thinking space:

28. Create a small dataset where mean, median and mode are all different.
Hint: Try five or six numbers with one repeat.

Your thinking space:

29. A student claims a 10-point increase from 10 to 20 is the same as from 90 to 100. Discuss percent change.
Hint: Compare relative change.

Your thinking space:

30. A school chooses the best house by total points, but houses have different numbers of students. Propose a fairer metric.
Hint: Use points per student or per event.

Your thinking space:

Notebook reflection. Which problem forced you to draw a diagram? Which problem had the most surprising idea? Write one mistake you made and how you fixed it.

Chapter 10

Functions, Growth, Optimization and Mixed Thinking

Classroom question: How can one rule describe change, and how do we choose the best option?

Learning goals

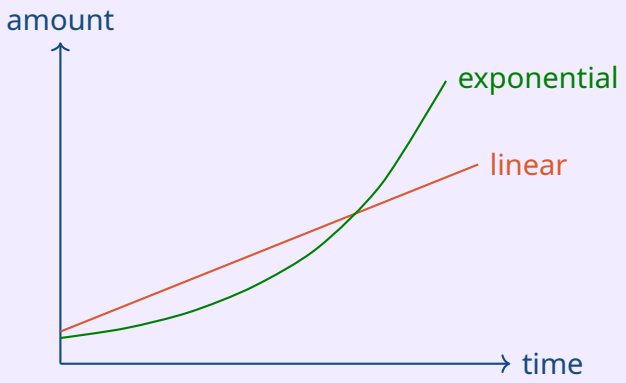
- Recognize functions as input-output rules.
- Compare linear, exponential and compound growth.
- Solve optimization problems with constraints.
- Combine arithmetic, algebra, geometry and data.

Theory and explanation basics

A function is a machine: one input goes in, one output comes out. If the rule is add 5, input 7 gives output 12. If the rule is double and add 1, input 7 gives output 15. The same idea appears in science, finance, spreadsheets, coding and graphs.

Growth can happen by addition or by multiplication. Linear growth adds the same amount each step. Exponential growth multiplies by the same factor. Optimization means choosing the best possible answer under rules: maximum area, minimum cost, quickest route or most efficient plan.

Visual idea



Linear adds the same amount. Exponential multiplies by the same factor.

Watch and learn further

- Dr. Pal playlist
- Dr. Pal - Optimization
- 3Blue1Brown - visual mathematics
- Khan Academy - functions

Two groups of online learning links

Group A: Dr. Biplab Pal class links

- Full Dr. Pal playlist - functions, optimization and applied math
- Dr. Pal - Inflation percent and growth
- Dr. Pal - How to use Math?
- Dr. Pal - Critical Reasoning

Group B: Other YouTube references

- YouTube search - Khan Academy functions middle school
- 3Blue1Brown YouTube channel - visual mathematical thinking
- YouTube search - optimization problems for middle school
- YouTube search - growth patterns math for kids

AI prompts for this chapter

Use these prompts after trying the ideas yourself. Ask for hints before full answers.

1. *"Explain a function as a machine with input, rule and output using a vending machine story."*
2. *"Give me a table, graph and sentence for this function rule: [paste rule]."*
3. *"Help me solve an optimization problem by making a table first, not by using calculus: [paste problem]."*
4. *"Create a challenging growth-pattern puzzle for me and give only a hint."*
5. *"Check whether my chosen solution is truly the best or only one possible solution: [paste work]."*

Solved classroom examples

Function machine
Rule: output = 3 times input minus 2. For input 5, output is $3 \times 5 - 2 = 13$.

Compound growth
Rs. 1000 grows by 10% each year. After one year: 1100. After two years: $1100 \times 1.10 = 1210$. It is not simply adding Rs. 100 forever if interest compounds.

Optimization rectangle
With perimeter 20 m, rectangles have $l + w = 10$. Areas: 1x9=9, 2x8=16, 3x7=21, 4x6=24, 5x5=25. The square gives the largest area among these whole-number options.

Exercises: 30 problems with hints

Easy practice: Problems 1-10

1. A function adds 4. What is output for input 9?
Hint: 9+4.

Your thinking space:

2. A function doubles. What input gives output 18?
Hint: Solve 2x=18.

Your thinking space:

3. A sequence starts 3 and adds 5 each step. Write first five terms.
Hint: Repeated addition.

Your thinking space:

4. A sequence starts 3 and doubles each step. Write first five terms.
Hint: Repeated multiplication.

Your thinking space:

5. Which grows faster at first: add 10 or multiply by 2 starting from 5?
Hint: Compare first few steps.

Your thinking space:

6. A rectangle has perimeter 20. If length is 6, find width.
Hint: 2l+2w=20.

Your thinking space:

7. Find 10% increase of Rs. 200.
Hint: 10% is one tenth.

Your thinking space:

8. If a machine rule is $y = x^2$, what is y for $x = 7$?
Hint: Square 7.

Your thinking space:

9. A pattern is 1, 4, 9, 16. What comes next?
Hint: Think square numbers.

Your thinking space:

10. Name one real-life situation where we want a maximum or minimum.
Hint: Cost, time, distance or area.

Your thinking space:

Classroom practice: Problems 11-20

11. A plant is 10 cm tall and grows 3 cm per week. Write a rule for height after w weeks.
Hint: Height = starting height + growth per week.

Your thinking space:

12. A bacteria count starts at 100 and doubles every hour. Find count after 5 hours.
Hint: Multiply by 2 five times.

Your thinking space:

13. Rs. 5000 is invested at 5% compound interest yearly. Find amount after 2 years.
Hint: Multiply by 1.05 twice.

Your thinking space:

14. A phone plan costs Rs. 200 plus Rs. 50 per GB. Write a function and find cost for 6 GB.
Hint: C(g)=200+50g.

Your thinking space:

15. A rectangle has area 48 and whole-number sides. List possible perimeters and find smallest.
Hint: Use factor pairs.

Your thinking space:

16. A school wants a rectangular garden with 24 m fencing. Compare areas for whole-number side lengths.
Hint: Use l+w=12.

Your thinking space:

17. A taxi company charges Rs. 30 start and Rs. 12/km. Another charges Rs. 60 start and Rs. 8/km. When is the second cheaper?
Hint: Set inequalities or compare costs.

Your thinking space:

18. A staircase has step heights 10, 12, 14, 16 cm. If pattern continues, find height of 10th step and total climb.
Hint: Arithmetic sequence and sum.

Your thinking space:

19. A YouTube channel has 1000 viewers and grows 20% each month. Estimate after 3 months.
Hint: Multiply by 1.2 each month.

Your thinking space:

20. A square and rectangle have same perimeter 32. Rectangle length is 10. Compare areas.
Hint: Find rectangle width and square side.

Your thinking space:

Super challenge: Problems 21-30

21. A farmer has 40 m fence for three sides of a rectangular pen against a wall. What dimensions give maximum area using whole meters?
Hint: If width is x, length is 40-2x; test values or build a table.

Your thinking space:

22. A rumor spreads: each person tells 3 new people per day. Starting with 1 person, how many new people hear it on day 5? How many total have heard by then?
Hint: Use powers of 3 and add levels.

Your thinking space:

23. Inflation is 6% per year. What raise is needed after 3 years to keep the same purchasing power?
Hint: Multiply by 1.06 three times.

Your thinking space:

24. A shop gives buy 2 get 1 free or 30% discount. For 6 identical items, which is better?
Hint: Compute effective price under each offer.

Your thinking space:

25. A rectangle with whole-number sides has area 60. Which dimensions minimize perimeter? Explain pattern.
Hint: Factor pairs closer together give smaller perimeter.

Your thinking space:

26. A function machine first doubles, then adds 3. Another first adds 3, then doubles. For what inputs are outputs equal?

Hint: Set $2x+3 = 2(x+3)$.

Your thinking space:

27. A town population grows by 10% yearly while water supply grows by 500 people-equivalent yearly. Create a starting value where supply eventually fails.
Hint: Compare multiplication with addition.

Your thinking space:

28. A path around a rectangular field has uniform width. Explain why area of path is not just perimeter times width for wider paths.
Hint: Corners overlap or add extra squares.

Your thinking space:

29. A student wants to maximize marks by choosing between easy and hard problems. Easy gives 3 marks and takes 2 minutes; hard gives 8 marks and takes 7 minutes. With 30 minutes and at most 6 hard problems, make a strategy.
Hint: Compare marks per minute and constraints.

Your thinking space:

30. Build a mixed problem using ratio, percent and geometry: a garden plan with a circular pond and rectangular flower beds. Solve your own problem fully.
Hint: State dimensions, compute areas and percentages.

Your thinking space:

Notebook reflection. Which problem forced you to draw a diagram? Which problem had the most surprising idea? Write one mistake you made and how you fixed it.

Final Mixed Project Problems

These projects combine several chapters. They are designed for group discussion, presentations and teacher-led workshops.

1. **Design a water-quality poster.** Use ppm and percent to explain a tank problem, include a diagram of sediment settling, and calculate how much contaminant remains after dilution. *Hint:* Track contaminant amount separately from water volume.

2. **Build a mini shop spreadsheet.** Choose five products, create prices, discounts and tax. Write formulas for final price and total bill. *Hint:* Each row should have original price, discount percent, discounted price, tax and final price.

3. **Create a number-system museum page.** Compare tally, Roman, decimal and binary by writing the same five numbers in each system. *Hint:* Include a paragraph explaining why decimal is convenient for humans and binary for computers.

4. **Measure pi at home.** Use three round objects, string and ruler. Make a table of circumference, diameter and ratio. *Hint:* Average the ratios and discuss measurement error.

5. **Plan a fair sports tournament.** Use ratios for team sizes, percentages for win rates, and statistics for ranking. *Hint:* Decide whether total wins or win percentage is more fair.

6. **Geometry treasure map.** Draw a coordinate grid, place three landmarks, and write distance clues using Pythagoras. *Hint:* Use right triangles between grid points.

7. **Budget under inflation.** A family budget has food, rent and transport. Apply different percent increases and decide what income raise is needed. *Hint:* Percent changes apply to each category separately.

8. **Optimization challenge.** With 36 meters of fencing, design the largest rectangular garden with whole-number side lengths. Then consider one side against a wall. *Hint:* Make a table of dimensions and areas.

9. **Data honesty poster.** Find or invent two graphs showing the same data, one fair

and one misleading. Explain the difference. *Hint:* Change the vertical axis or scale to see how impressions change.

10. **Teach one concept.** Choose fractions, equations, pi or graphs. Create a one-page lesson with one visual, one solved example and three exercises. *Hint:* A good explanation uses both words and pictures.

Closing Note

The goal of this book is not to finish pages quickly. The goal is to build habits: draw, translate, estimate, solve, check and explain. A student who can explain a wrong attempt clearly is already doing real mathematics.

About the Author

Dr. Pal is a technology entrepreneur, adjunct professor at the University of Maryland Baltimore County, and a passionate educationist who has established several educational initiatives across continents. He holds more than 30 US patents in IoT and AI, and has written more than 30 Math and Physics question books in the Poccomu Series for middle- and high-school students. He has many publications in the areas of AI, autonomy and telecommunication.

To get a free PDF or printed version of this book or any other book by Dr. Pal, please send a WhatsApp message.

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